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Report on the IUTAM symposium: fundamental aspects of vortex motion

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The IUTAM Symposium *Fundamental Aspects of Vortex Motion* was held in Tokyo, Japan, from 31 August to 4 September 1987. We present an account of the technical sessions of that meeting. The main goals of this report are (i) to provide a widely accessible record of the four-and-one-half day meeting; (ii) to identify important new developments in the field of vortex dynamics of potential interest to a larger audience than the invited attendees; and (iii) to attempt some overview comments with the wisdom of hindsight that may be useful as a guide to specific papers in the proceedings and other current literature.

1. Introduction

The identification of *vortex motion* as a subtopic within fluid mechanics can be traced rather precisely to the seminal 1858 paper of Helmholtz *Über Integrale der hydrodynamischen Gleichungen, welche den Wirbelbewegungen entsprechen*. Since then the literature on this subtopic has grown steadily, much of it acquiring a distinct flavour due to the dominant role played by Lagrangian concepts in vortex dynamics. Flow visualization techniques, these days employing laser light and fluorescent dyes, produce stunning pictures of vortex patterns†. And a whole class of numerical methods have sprung up in which the Lagrangian vorticity is tracked instead of discretizing Eulerian fields onto grids fixed in space. Both of these important developments derive from Helmholtz's powerful insight that (in Tait's 1867 translation) '[e]ach vortex-line remains continually composed of the same elements of fluid, and swims forward with them in the fluid'.

The International Union of Theoretical and Applied Mechanics (IUTAM) recently sponsored a symposium on the subject of vortex dynamics. The title was *Fundamental Aspects of Vortex Motion*. The Symposium was held from 31 August to 4 September 1987 in the Congress Hall of the Science Council of Japan (Nihon Gaku-Jutsu Kaigi) in Tokyo, Japan. Professor H. Hasimoto chaired the International Scientific Committee, and Professor I. Imai was chairman of the Local Organizing Committee. One of the authors of this report (H. A.) was a member of the International Scientific Committee, the other (T. K.) was secretary of the Local Organizing Committee.

† Since such flows are invariably unsteady, the *interpretation* of dye lines or patches must be done with care. Also the tendency to *equate* dye location to vorticity location must be resisted since the diffusivity of the former can be controlled, while the diffusivity of the latter is the kinematic viscosity, which typically is several orders of magnitude larger.

This is not the first time the subtopic of vortex motion has been recognized by IUTAM with a symposium. Küchemann (1965) reported on an earlier symposium entitled *Concentrated Vortex Motions in Fluids* some two decades ago. There have, of course, been many meetings sponsored and organized under other auspices. The symposia are particularly attractive in bringing together a broad cross-section of workers from theoretical, experimental and computational backgrounds.

Vortex dynamics is a rapidly evolving part of fluid mechanics. It touches on virtually every other branch of the field. However, papers in vortex dynamics tend to focus on mechanistic and deterministic aspects of the flow. As such, vortex dynamics provides a fresh approach to issues in turbulent flows, and a link to developments in other subjects such as the theory of dynamical systems and topology. The present Symposium reflected these trends.

Most of the papers presented at the Symposium are scheduled to appear in a proceedings volume that will be a special issue of the journal *Fluid Dynamics Research* (vol. 3, 1988). Our intention in writing this report is to provide a synopsis of the Symposium capturing some of the pervading themes and ideas, which are inevitably de-emphasized in a collection of technical papers, especially since the papers were due at the Symposium and cross-referencing of work within the Symposium was virtually non-existent. In writing the report we have not adhered to the chronological order in which the papers were presented. We have taken the liberty of adding commentary on problems, ideas and trends in the field. Some of these are our own reactions and rationalizations. Others transpired in discussions, questions and comments during the Symposium. Finally, a few colleagues who attended have kindly looked over a preliminary version of this report and given us further input.

We hope the report will be found useful to those who did not attend the Symposium (and maybe to some of those who did). In particular, we hope it will be useful to newcomers to the field of vortex dynamics, and to those who cannot find time to savour the proceedings volume in its entirety.

2. The vortices

The first step in a theoretical discussion, a numerical simulation or even in interpreting data from an experiment is often to decide on a suitable model of the vortex or vortices in question. Indeed, considerable debate can arise in deciding which model one should favour. Concentrating the vorticity on curves or points is attractive analytically for it leads to a dramatic simplification of the dynamical equations. Several models of this general type are in current use, and more or less elaborate theoretical constructs surround each one. Hasimoto*† started off the technical sessions by giving a broad overview of vortex motion including such issues.

Starting with the simplest case, we have in two dimensions the concept of a *point vortex*. As has been known for at least a century the motion of point vortices is accessible via the formalism of classical Hamiltonian mechanics. In three dimensions the corresponding concept is the *vortex filament*. Again a reduction in dimensionality by two is achieved, but the vortex filament is a one-dimensional *continuum*. The dominant term in the motion of a curved filament is a local one – proportional to the curvature and directed along the binormal – and retention of just this contribution

† An asterisk to a name indicates a paper presented at the Symposium. All such papers are listed in the References.

leads to the very pretty theory known as the *localized induction approximation* or *LIA*. The history of this approximation, which goes back at least to the paper by Da Rios (1906), was traced by Hama*. A further conceptual reduction to little 'vortex points', usually referred to as *vortons*, leads to various models of three-dimensional flow. This class of models was discussed by Novikov* and Kuwabara*. Some controversy still surrounds this representation (cf. Saffman & Meiron 1986). Filaments may be thought of as chains of vortons, much as vortex sheets in two dimensions may be approximated by rows of point vortices.

More general distributions of vorticity, in which the vortex retains a finite core, have been introduced in two dimensions, and for the closely related case of three-dimensional axisymmetric flow. In the simplest case the vorticity has a 'top hat' profile, constant within certain contours, vanishing outside. Such regions are called *vortex patches*. Extensions to several nested contours are possible. Their evolution under mutual induction leads to integro-differential equations usually referred to as *contour dynamics* (Zabusky, Hughes & Roberts 1979).

Another idealization is the concept of a *vortex sheet*. These are surface or line singularities, that have an intrinsic instability, named for Kelvin and Helmholtz, due to the tangential discontinuity in velocity. Two discussions of vortex sheets were given, both from a mathematical vantage point. Krasny* discussed a smoothing technique in which the denominator in the Birkhoff-Rott integral is desingularized by adding a term δ^2, where the adjustable parameter δ gradually tends to zero. In this way he hoped to study the nature of the singularity that forms after a finite time, as expected on the basis of an asymptotic analysis by Moore (1979) and numerical results of Meiron, Baker & Orszag (1982). An intriguing mathematical problem is what happens right at the singularity time? The prevailing idea is that a spiral occurs instantaneously. Caflisch* discussed rigorous results that he and Orellana had obtained relating to the interval from $t = 0$ to the time of appearance of the singularity. He made use of the notion introduced by Moore (1979, 1984) of analytic continuation to complex values of the vortex-sheet strength. There was little discussion of the physical relevance of these results.

All these model vortices impose a reduction in the number of degrees of freedom necessary for the description relative to the general problem. To capture what an entirely general distribution of vorticity will do we must usually resort to flow visualization or to detailed numerical simulations. As we increase generality, we typically loose analytical tractability, at least in the traditional sense of explicit formulae. Tools from modern mathematics may come to the rescue here. To make progress we may have to think in terms of broader characterizations via the symmetries and topology of the problem rather than in terms of configurations known in complete detail. For example, Moffatt* described how his method of magnetohydrodynamic (MHD) relaxation (see Moffatt 1986) could be used to ensure the existence of an entire class of steady vortex rings (and vortex pairs). Hill's vortex is one member of this family, but the theory establishes the existence of a vortex in this class under very general conditions. There is a restriction. Only flow topologies such as figure 2(*a*) can be handled. The topology of figure 1(*b*) has a troublesome saddle-type stagnation point and the relaxation method can introduce an undesirable vortex sheet. There may, unfortunately, be many configurations that one cannot relax to by this method: the point-vortex equilibria considered recently by Campbell & Kadtke (1987) might be an example, and for states with several vortex patches the relaxation again appears to produce unwanted vortex sheets. On the other hand, piecewise uniform vortex regions *bounded by vortex sheets*, so-called *Sadovskii vortices*

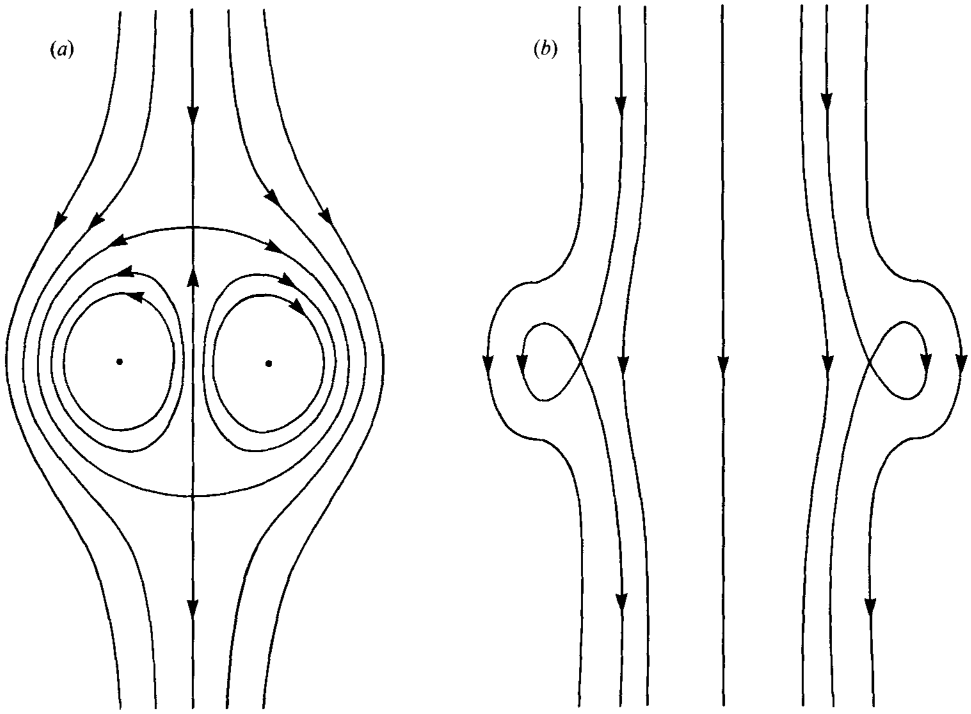

FIGURE 1. Streamline topologies for translating vortex pairs or rings. Configurations of the type shown on the left (*a*) are amenable to the MHD relaxation technique of Moffatt* and yield smooth solutions of the Euler equations. Configurations of the type shown on the right (*b*) relax to solutions with embedded vortex sheets.

(cf. Smith 1985) that might arise as models of wakes behind bluff bodies in the limit of infinite Reynolds number, are potentially accessible by this method. The technique is still evolving.

Another result of similar, general flavour was mentioned by Saffman* in his survey lecture. This is the remarkable implications for linear stability of fore–aft symmetry for vortex-street configurations, recently found by Jimenez (1987) and Mackay. The topology of the stability diagram survives a change of the vorticity distribution in the street vortices if the fore–aft symmetry is preserved.

Saffman* also stressed the *degeneracy* that one observes in inviscid flows. He cited as an example what is known about steadily rotating vortex patches: the 'Rankine vortex' (a circle), the 'Kirchhoff vortex' (an ellipse), and then the 'triangles', 'squares', etc. found by Deem & Zabusky (1978). These can all be thought of as bifurcations from the circle. Saffman* mentioned new results on further bifurcations from the elliptical branch leading to vortices that rotate without change of shape but have only a single axis of symmetry. He speculated that such bifurcations might go on hierarchically as the parameters were changed, ultimately resulting in solutions to the Euler equations with highly ramified (fractal?) boundaries. Viscosity (or other physical effects) would presumably select from these degenerate, inviscid families certain members that are realized in high Reynolds number flows.

Vortex motion in compressible flows is quickly becoming an intensely studied topic. Pullin & Moore* reported analysis of one of the simplest possible cases: a vortex pair propagating through an ideal gas. The vortices had a hollow core in the sense of Pocklington (1895), and a major part of the problem was to determine their shape. A hodograph method was used for the analysis. Surprisingly, solutions with smooth boundaries could not be found even at very low values of the Mach number at infinity. The flow, although subsonic in the far field, would become supersonic close to the vortices. Addition of a shock wave bridging the vortices seemed to be essential to match the solutions to experimental observations. A video by Chalmers *et al.** showed the vortices generated when a shock wave passes across a bubble containing gas of a different density. These are axisymmetric analogues of the states investigated by Pullin & Moore*. The video was from a computer simulation but many thought it was of a laboratory experiment. Zabusky, who presented the video, noted that there was qualitative agreement between the simulations and recent laboratory results on the same problem by Haas & Sturtevant (1987), although these were not shown.

Many of the vortices considered at the Symposium were (or were considered to have been) produced by shedding from solid bodies. We shall deal with this class of phenomena in §4. Vortices can also arise from instability mechanisms in steady flows where, upon change of some control parameter, distributed vorticity is focused into coherent structures. The vortices producing 'secondary flows' are typically of this kind. Ohji* had studied the regime of modulated wavy vortices in a Taylor–Couette flow system. The flow visualization employed a mirror shaped as a collar about the apparatus so that the entire 360° of the pattern could be visualized simultaneously. Niino* studied the vortices produced in a spin-down experiment shown at the session, and explored transitions from regular to irregular patterns. These are presumably Görtler vortices (Weidman 1976). In another demonstration experiment Takematsu & Kita* showed how to produce monopolar eddies in a rotating tank by thermal forcing (local cooling by ice). Such eddies are often used as laboratory analogues of Gulf Stream rings (see, for example, Capéran *et al.* 1988). Noto, Honda & Matsumoto* showed experimental results on the vortex motion generated in a thermal plume pushing through a stably stratified environment, and considered these in the context of coherent structures in a turbulent flow. A similar point of view was adopted by Tokunaga, Satofuka & Itinose* who reported on numerical simulations of steadily propagating eddies of fixed form in plane channel flow.

3. Modes and mechanisms

As indicated in the Introduction one of the fascinations of vortex dynamics is that it allows a mechanistic Lagrangian description of the motion of fluids. The idea of induction as embodied in the Biot–Savart integral is central to such developments. A wide spectrum of phenomena is possible. In two dimensions point vortices can participate in *chaotic motions* as discussed by Y. Kimura* for vortices in bounded domains (see also Hasimoto *et al.* 1984) and by Aref *et al.** for collisions of vortex pairs (see also Eckhardt & Aref 1988). Aref *et al.** also discussed steadily moving configurations of point vortices (an issue in the topic of *vortex statics* as Kelvin called it). They presented eight 'open problems' concerning the aspects of point-vortex motion discussed. In three dimensions filaments with a finite core can support *solitary waves*, and in the idealized description of LIA even *solitons* (Hasimoto 1972). The single soliton solution of the nonlinear Schrödinger equation (NLSE) was part of the logo for the Symposium. It is a problem of continuing interest to see to what extent features discovered for singular vorticity distributions carry over to the more realistic dynamics of distributed vorticity.

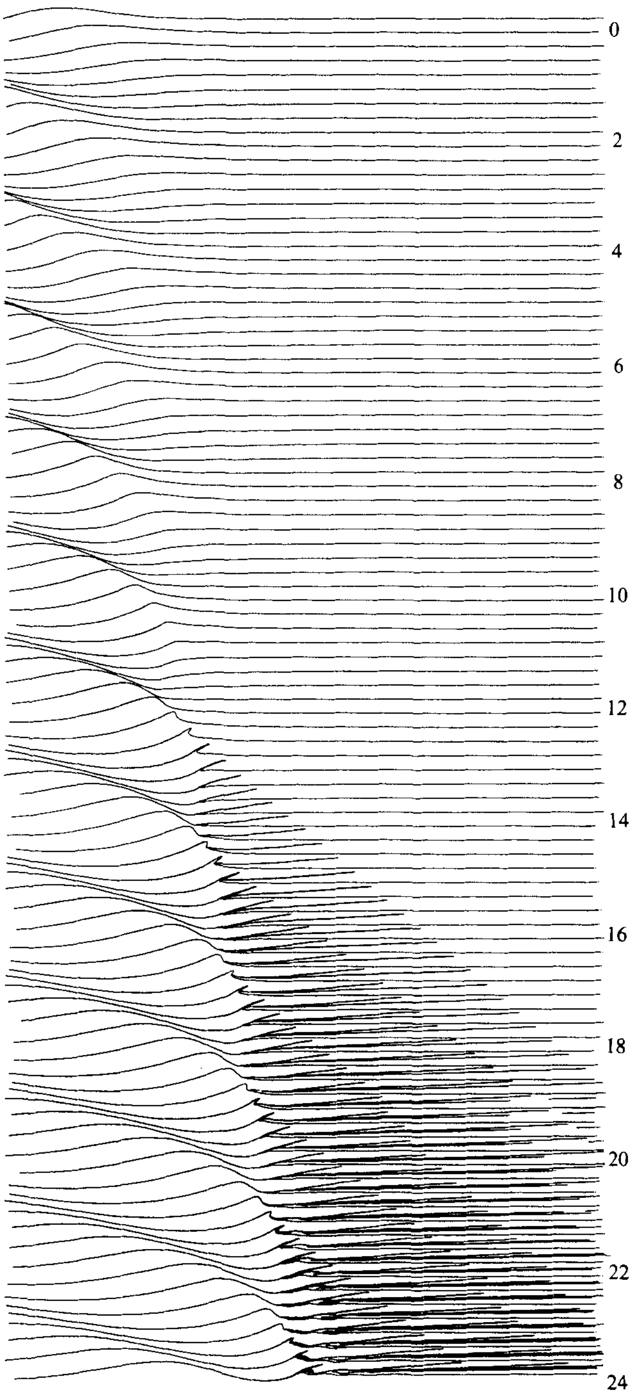

FIGURE 2. Simulations of repeated filamentation at the rim of a circular vortex patch by Dritschel*. Each line corresponds to a small part of the vortex boundary (across which the vorticity jumps discontinuously) drawn in polar coordinates. Evolution proceeds from top to bottom. Snapshots are spaced apart in time by one-eighth of the period of the undular motion of the interface. The nearly linear, periodic behaviour of the imposed disturbance observed early in the calculation eventually gives way to filamentation. The boundary then rapidly grows in complexity, not only from the nearly periodic generation of filaments, but also because filaments subsequently induce new filaments.

Zabusky suggested that 'vorticity is loose and soft', not just in the sense of an elastic rod supporting a propagating twist, but particularly in the dynamics of vortex core deformation. A considerable vocabulary has already been built up regarding this problem in two-dimensional flow: like-signed vortices can *merge* and the waves on their boundaries can break (indeed repeatedly; see below), a process known as *filamentation* (Christiansen & Zabusky 1973). For vortices of opposite sign the production of translating pairs is often referred to as *coupling*. In this case any excess vortical fluid is left behind by *spiking*. Using this jargon filamentation is to merging what spiking is to coupling. In an external straining field single vortices may be elongated and split into two or more pieces, leading to the mechanism of *tearing* (Moore & Saffman 1975) as the 'opposite' of merging.

Several examples of these mechanisms were shown: Dritschel* produced numerical simulations of repeated filamentation at the surface of a vortex patch (see figure 2). A ciné film of this process was also shown. He suggested that filamentation must occur generically, and pointed out that it is quite consistent with bounds obtained in the theory of nonlinear stability of vortex patches (Wan & Pulvirenti 1985; Dritschel 1988*a*). Again, vortices with highly ramified boundaries were suggested, this time as the result of dynamical evolution under the two-dimensional Euler equations instead of repeated bifurcations of steady states. Shariff *et al.** showed simulations of Hill's vortex in which deformations lead to spikes (see also Pozrikidis 1986). In this case the flow is linearly unstable. Capéran & Verron* had performed numerical simulations to elucidate apparent contradictions between existing computations of merging criteria and earlier experiments (see Capéran *et al.* 1988). Questions from the audience indicated that there were still controversial points here. Chollet, Lesieur & Comte* studied the vortex interactions, mainly pairings, seen in two-dimensional mixing layers and jets, and also displayed results for an advected and diffusing passive-scalar field. Polvani, Zabusky & Flierl* reported extensions of contour dynamics to a two-layer model with quasi-geostrophic flow. One now has new possibilities such as two-layer merger. Discrepancies between the simulations and recent experiments of Griffiths & Hopfinger (1987) were noted. Farge* showed simulations using the shallow-water equations, and interpreted the spiking and filamentation in terms of a 'direct cascade' to smaller scales. The merging represents an 'inverse cascade' to larger scales. This *qualitative* argument has been made for many years. With the increasingly detailed understanding of the elemental processes it would be interesting to produce a quantitative theory based on the observed vortex dynamics.

In three dimensions there are clearly additional possibilities for the evolution and interaction of vortices. Saffman* outlined the theory of small-amplitude instabilities for a vortex filament in an external strain. Three regimes may be identified depending on the relative magnitude of the wavelength of the perturbing wave, λ, and the vortex core radius a. When $\lambda \gg a$ the Biot–Savart law determines the dispersion relation. For $\lambda \approx a$ there is a parametric wave instability regime first described by Widnall, Bliss & Tsai (1974). Finally, for $\lambda \ll a$ Saffman* called attention to the recent work by Pierrehumbert (1986) and Bayly (1986). He stressed that the growth rate for all three classes of instabilities is of the order of the external rate of strain. The main distinction between regimes is in the wavelength range.

Considerable attention attaches to the process of *reconnection* where, owing to weak viscous dissipation, the topology of an initial vortex configuration can change. Kida & Takaoka* and Melander & Zabusky* presented full numerical simulations of this phenomenon (see figures 3 and 4). The former paper considered the helical flow

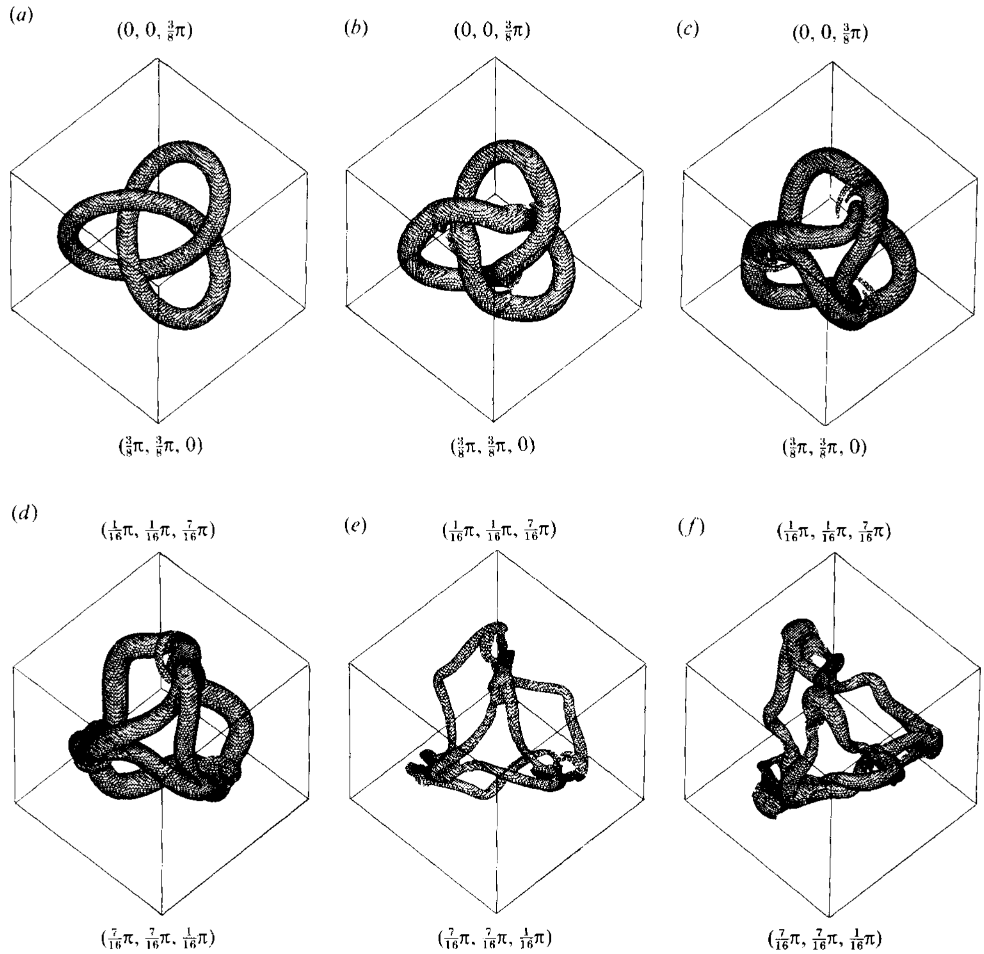

FIGURE 3. Simulation of vortex reconnection by Kida & Takaoka*. The initial condition (*a*) is a trefoil knot. The computational cube has side $\frac{1}{2}\pi$, but a smaller cube of side $\frac{3}{8}\pi$ is shown in all panels. Since the configuration translates as a whole, the same cube is not shown in all panels, so two corners are labelled. The vortex is always viewed from the point (3,3,5). Surfaces $|\omega| = \text{const.}$ are shown at various times t, where (*a*) $t = 0$, $|\omega| = 31.5$; (*b*) 0.1, 27.6; (*c*) 0.2, 27.8; (*d*) 0.3, 32.3; (*e*) 0.4, 89.6; (*f*) 0.5, 19.7. The Reynolds number based on vortex circulation and kinematic viscosity is $\Gamma/\nu = 800$.

about a trefoil knot known to be a steadily translating state within LIA (Kida 1981). The latter paper considered an initial state consisting of two rectilinear vortex tubes at right angles. (A scheduled paper by Ashurst, Meiron, Orszag & Shelley, containing further numerical simulations of reconnection, was unfortunately not presented.) In both of the calculations dramatic deformations of the vortex cores result, leading to highly three-dimensional interconnection patterns variously termed *fingering* and *bridging*. This topic has seen rapid development since the Biot–Savart calculations with a simple core model by Siggia (1985) and the elaborations of this work by Pumir & Siggia (1987). In this earlier work, and closely related work by Schwarz (1982, 1985) in the literature on turbulence in superfluid ^{4}He, the details of core deformation are not captured by the numerical methods. The observation in this work that just before reconnection the two vortex cores become antiparallel is, of course, borne out

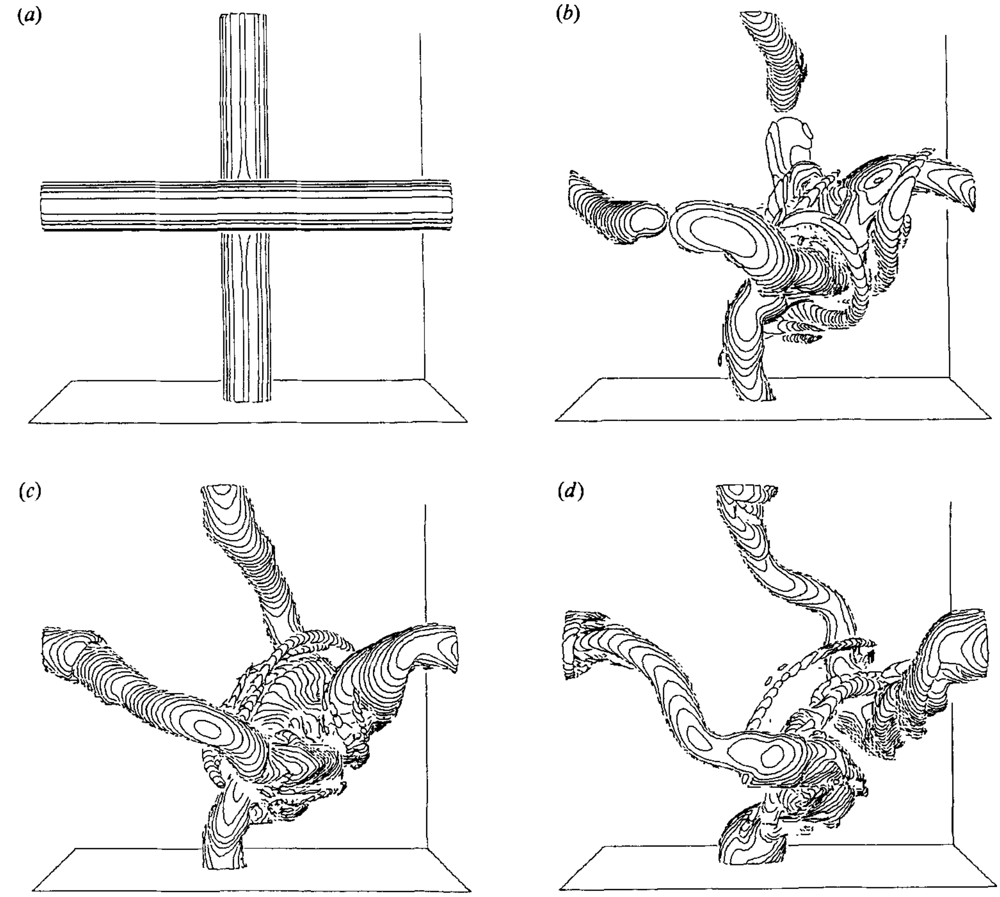

FIGURE 4. Simulation of vortex reconnection by Melander & Zabusky.* Two vortex tubes at right angles constitute the initial condition (*a*). As in figure 4 surfaces $|\omega| = \text{const.}$ are shown at various times t. Here $|\omega| = 16$ in all panels and (*a*) $t = 0$; (*b*) 1.5; (*c*) 2.0; (*d*) 2.5. The Reynolds number based on vortex circulation and kinematic viscosity is $\Gamma/\nu = 8 \times 10^5$.

by the recent, more detailed computations. Zabusky offered the synopsis that 'reconnection' is to some extent a misnomer. Only an *apparent* reconnection occurs in flow visualization pictures. In reality a complex rearrangement of vortex lines takes place and the observed reconnection results from a 'coarse-graining' of the vortex tangle due to the flow visualization techniques. It may be said to happen at the instant when the fingers or bridges are completed. Considering the amount of detail available for two-dimensional merging, it would appear that we have only scraped the surface of the more complex three-dimensional problem. Takaki & Hussain* described attempts to tackle the problem analytically by expanding the velocity field in a power series and keeping only the lowest non-trivial terms (see also Takaki & Hussain 1985). Their physical picture of the process was based on the observation that two filaments connected in two different ways differ by a ring vortex (see figure 5). This piece of insight, which they attributed to the superfluid ^{4}He literature, seems very interesting, but the complexity of the numerical simulations suggests that more than low-order polynomials will be required in a full quantitative theory. The vortex ring halo that one introduces in the model

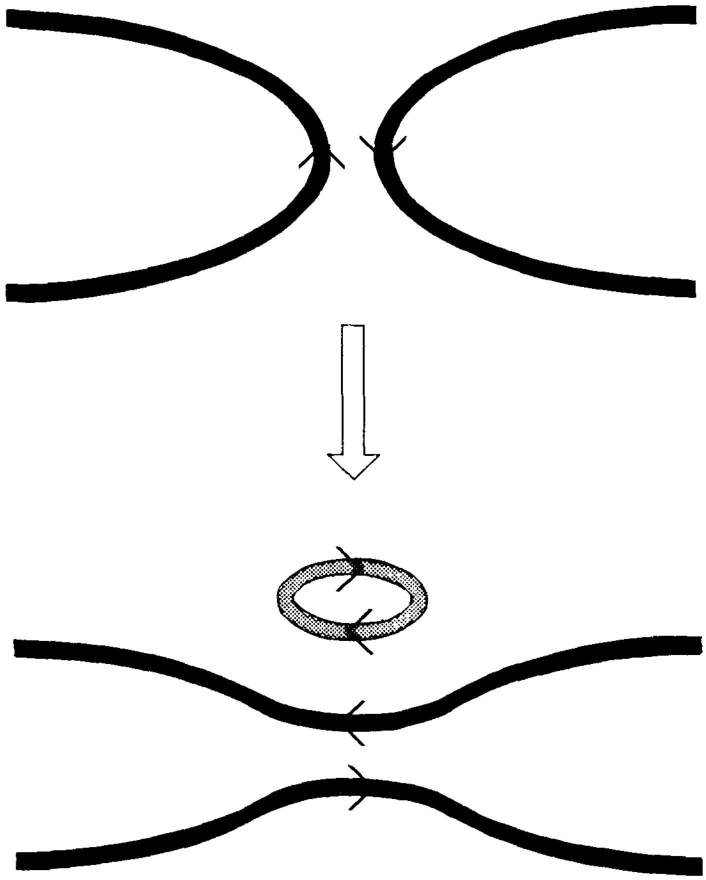

FIGURE 5. The idealized reconnection model investigated by Takaki & Hussain*. The two reconnections differ by a vortex ring as indicated. The open arrow indicates evolution from the configuration at the top to that at the bottom.

representation of figure 5 may in fact be an idealization of the 'dipolar structure' formed by fingers and bridges between the original filaments in the full simulations (see in particular panels (*b*) and (*d*) of figure 4).

Related to these studies was the work by Oshima *et al.** in which bifurcations of an elliptical vortex ring were studied. Many years ago Kambe & Takao (1971) and Oshima & Asaka (1977) showed experimental results in which two vortex rings travelling side by side attach to form an elongated elliptical vortex which then oscillates and breaks along a plane rotated 90° relative to the plane of initial osculation. When the rings are coloured differently, the two new rings consist of different coloured halves. (A scheduled paper describing recent work on the two-ring problem by Coles & Schatzle was unfortunately not presented.) The natural extension of this work is to start with an elliptical vortex, as did Oshima *et al.**, thus focusing on the latter half of the process. Clearly, this is a realization of reconnection, but most of the detail seen in the numerical work is so far without experimental verification. Kiya & Ishii* considered the problem of a vortex ring interacting with an initially straight vortex filament using a computer simulation method of the vorton type. This problem may be thought of as a limit of the two-ring problem in which the radius of one ring has become arbitrarily large. Although fine details of the reconnection were not resolved by the numerical method used, the calculations of these authors suggested an interesting transition from a *direct* scattering regime, where the ring remains intact as it passes by the line, to an *exchange* scattering regime, where a piece of the ring changes places with a piece of the line, and a ring made up partly of new vortex fluid (from what was originally the line) emerges. This is not inconsistent with the experimental results for two-ring collisions mentioned above, but the line-ring collision experiment is more difficult, and has not so far been done. At the cross-over from the direct scattering regime to the exchange scattering regimes a messy *coalescence* or *trapping* of the ring by the line seemed to take place in the computations. It is interesting to note that analogous transitions from direct to exchange scattering scenarios occur also for interactions of a point-vortex pair with a single vortex (Aref 1983). It would seem worthwhile to subject the line–ring collision process in three dimensions to the higher resolution numerical experiments reported by other authors.

The evolution of vortex configurations in time in a fixed spatial domain has as its counterpart the evolution with a spatial coordinate of the vorticity in a steady flow. Prime among these is the problem of *vortex breakdown*, a very active topic when Küchemann wrote his report in 1965 and still going strong. Keller, Egli & Althaus* gave a systematic classification of the various possible flow regimes and their interrelations in tubes with varying cross-section. The analogies to gravity currents in hydraulics and to steady, quasi-one-dimensional gas dynamics were elaborated. Maxworthy* in a general lecture on waves propagating along vortex cores pursued the model of an axisymmetric wave pushing upstream against a mean flow. He argued that the instability of this state would lead to spiral waves, and that the transition between the 'bubble' and 'spiral' types of breakdown really is a continuous one depending on how much the spiral instability has degraded the axisymmetric 'bubble'. Another mechanistic explanation suggested was the so-called 'skipping effect' for tornadoes or dust devils during which they leave the ground for a certain distance. This, Maxworthy suggested, could be rationalized by considering an axisymmetric wave approaching the ground and then being reflected. During such a process the base of the tornado would first widen dramatically, and then suddenly thin, possibly producing a detachment of the vortex from the ground. A type of vortex breakdown in the swirling flow above a sinkhole, in which a transition from a one-celled to a two-celled vortex is observed, was discussed by Shingubara *et al.** And Krause* reported on attempts at quantitative prediction of vortex breakdown using full Navier–Stokes simulations.

Another interesting topic covered by Maxworthy's* survey was the issue of agreement between theory and experiment on the propagation of helicoidal solitary waves along vortex cores. Since the experiments some years ago by Hopfinger & Browand (1982) and by Maxworthy, Hopfinger & Redekopp (1985) there have been discrepancies with the available theories. Whereas the axisymmetric waves tend to be governed by equations of KdV type, irrespective of the core structure, the helicoidal or bending waves invoke the NLSE and variations of it. Axisymmetric waves are by far the most common, accounting for some 70 % of all waves observed in a general 'turbulent' setting. The bending waves are only seen some 10–15 % of the time unless special procedures are used to initiate them. The shapes of the 'Hasimoto soliton' seem adequate to fit experimental data, but the ratio between group velocity and phase velocity for the wave is a constant (2.0) in LIA, whereas the experiments show a dependence on the ratio of wavelength to core size. Also unexplained is a large phase advance on collision of two experimental 'solitons'.

An intriguing attempt to go beyond LIA as originally conceived was presented by Fukumoto & Miyazaki*. Their proposal to include an arbitrary axial flow led after lengthy analysis and a complicated matching procedure to a dynamics that could be transformed via Hasimoto's (1972) procedure into the Hirota equation rather than the NLSE. This leaves the shape of the solitons unchanged, changes the velocity of propagation, but, unfortunately, does nothing for the troublesome phase jump. Discrepancies remain, but the new model deserves further exploration. In their analysis Fukumoto & Miyazaki* claimed to differ with a similar analysis by Moore & Saffman (1972). That point is certain to be scrutinized. In order to obtain the experimental observations on solitary waves on vortices, and their interactions, there may be no way around the full, finite-core problem, i.e. all filament models with a simple description of the core may be inadequate. For recent important contributions on the full problem see Leibovich, Brown & Patel (1986). Other aspects of the Hopfinger & Browand (1982) experiment, notably the influence of rotation on the turbulence generated at the bottom, were examined by Mory*.

Solitary waves apparently exist also on the cores of vortices in superfluid ^{4}He. Maxworthy* reported experiments on inelastic neutron scattering off vortices in thin films of the superfluid. Using a hollow-core model (for want of something better) and assuming that helical modes could be excited (along with the well-established rotons and ripplons) close agreement was obtained in a dispersion relation plot.

Commenting in the final, informal discussion on the simulation results of Kida & Takaoka*, Moffatt emphasized their observation that the helicity of the flow remained rather constant while the energy decreased. He reminded us of the Cauchy–Schwarz inequality between helicity, energy and enstrophy (Moffatt 1969), and that the flow for which equality is attained is a Beltrami flow, thus fuelling the recent notion of *Beltramization* of (the large scales of) turbulence. This discussion tied in with results of Kit *et al.**, reported by Tsinober, in which the vorticity and velocity were measured in two seemingly independent realizations of grid turbulence: a salt-water flow in Tel-Aviv and an air flow in College Park, Maryland. Without apparently injecting any helicity into the flow it was found in both cases that the turbulence lacked reflection symmetry. Furthermore, the correlation coefficient between a component of velocity and the same component of vorticity were of the same order of magnitude in the two experiments but of opposite sign! A lively discussion ensued, in particular since the last paper of the Symposium by Frisch *et al.** dealt with a new predicted effect, that in a turbulent flow would lead to exponential growth of large-scale modes satisfying the Beltrami condition, given only that the flow was not 'parity invariant' at the outset. This 'AKA-effect' is an analogue for incompressible flow of the so called α-effect in MHD. The discussion centred on whether Kit *et al.** were really observing the 'Beltrami runaway' that could occur via the AKA-effect. Rather deep questions about the accuracy to which one needs to know initial conditions in an experiment were raised.

4. The role of solid boundaries

Producing vortices in ordinary fluids at high Reynolds number invariably involves the complex process of *shedding*. In the simplest case this is a time-periodic response to a steady external condition (the uniform oncoming flow). There are several interesting problem areas associated with this phenomenon.

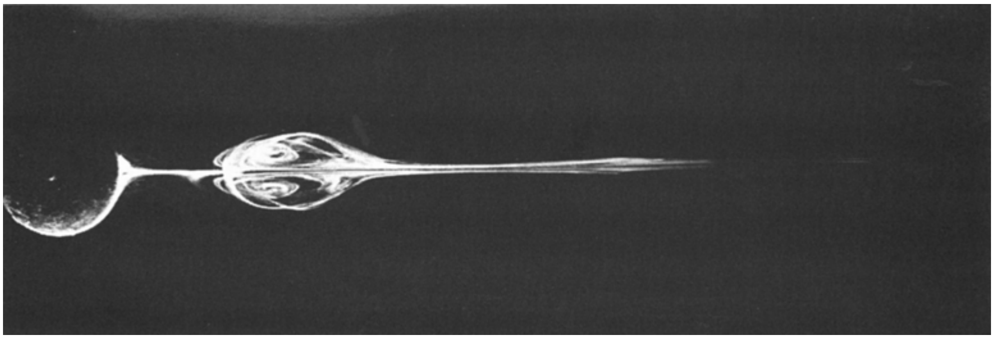

FIGURE 6. Example of a detached vortex 'bubble' wake downsteam of a sphere moving in a linearly stratified fluid as visualized by Honji*. The Reynolds number based on mean flow speed U, kinematic viscosity ν, and sphere diameter D is $Re = UD/\nu = 287$. The Brunt–Väisälä frequency is $N = 2.3U/D$. The fluid depth was 20 cm., the sphere radius 3.19 cm.

The most obvious is to study the vortex motion produced by the shedding at large distances from the body. This includes such important topics as the vortex system associated with a wing in flight. Roll-up in the Trefftz plane is a classical problem here. Numerical simulations were presented by Krasny* who considered the effect of the mathematical singularities at the edges of the finite-span vortex sheet (see Krasny 1987).

There were several more physically oriented studies of the separation vortices on delta wings, a problem of long standing that is prominent in Küchemann's (1965) report as well. Nastase* and Schmücker & Gersten* reported on experimental results aimed at controlling the vortex and postponing breakdown. A major effort known as the *International Vortex flow Experiment* is underway to address this topic. Hornung & Elsenaar* gave a synopsis of this work. The issue of prediction of the vortex motion via numerical codes aimed at solving Euler's equation was addressed, and summarized with a quote of J. H. B. Smith: 'Computational methods are still developing rapidly – and we can be glad about that, for they have some way to go – but the stock of good [experimental] data is not large. Moreover, good data last forever.'

The dynamics of shedding off solid bodies of other shapes were investigated by several authors. Okude & Matsui* had performed experiments on vortex-street formation behind a flat plate parallel to the flow. Bearman & Takamoto* had studied the shedding by a ring. For a very thick ring the shedding approaches that of a solid disk. For a thin ring the shedding approaches that of a strip, i.e. one expects independent Kármán streets from opposite points on the ring. The interest was in the cross-over from one kind of wake to the other. Wei & Lin* had studied the shedding patterns and their associated forces on a seamless, non-rotating volleyball (see in this connection the recent review by Mehta 1985).

Honji* showed flow visualizations of the wake produced behind cylinders and spheres in stratified fluid. The relation between Strouhal number and Reynolds number now depends on the Brunt–Väisälä frequency. A comprehensive parametric study was presented with some intriguing 'detached bubble' wakes for certain choices of the parameters (figure 6). Keller commented that many of Honji's pictures looked surprisingly like known vortex breakdown regimes, and an analogy between stratification and centrifugal force was suggested.

R. Kimura* discussed the spectacular shedding and formation of a vortex street that one sees in cloud patterns around Cheju island. In a particular 24 hour sequence on 2 April 1987 the 400 km long trail seemed to oscillate with vortices of opposite sign on the two sides of the street alternately amplifying and subsiding.

An analytical paper by Wu, Wu & Wu* discussed the general expressions for calculating the force on a moving body in a viscous, compressible fluid.

Several authors were concerned with the numerical modelling of the shedding process. For a body with sharp corners a common assumption is that the shedding point is known (although this is an approximation), and 'vortex methods', where an elemental vortex is released at each time step, have been tried by many. Graham & Cozens* showed simulation results and comparison flow visualizations for sharp and slightly rounded 90° corners. They used the vortex-in-cell method (Christiansen 1973) modified to take viscous diffusion into account. (Right-angled corners fit nicely on the underlying square grid typically used by this method.) Closely related simulation results were shown by Faltinsen & Braathen* for two-dimensional floating bodies, i.e. for flow with a free surface. Both investigations were aimed at addressing roll-damping of ship hulls and flow-structure interactions on off-shore structures. A couple of the main conclusions were: first, that the introduction of viscosity leads to a secondary separation relative to the single vortex separation seen in inviscid calculations with a Kutta–Joukowski condition; second, the curvature of the corner is a crucial parameter; finally, the damping from eddy generation and that from surface wave motions seemed to be closely linked effects. High-resolution, grid-based simulations of shedding from square cylinders at various angles of attack and Reynolds numbers of order 10^5 were presented by Shirayama, Kuwahara & Tamura.* These calculations were apparently intended to model the wakes of skyscrapers in modern Tokyo. Soh, Hourigan & Thompson* presented a model claimed to capture shedding from smooth surfaces.

Auerbach* described the generally sad state of our understanding of the formation of vortex rings and pairs by pushing fluid out of a tube or nozzle. There are several parameters: Reynolds number, nozzle cone angle, ratio of nozzle opening to tube diameter, and fluid stroke. The angle at which a nascent pair propagates relative to the nozzle wall, its speed of propagation and the ratio of tube fluid to ambient fluid found within the vortex all depend sensitively on these parameters. Many of the observed relationships are not understood theoretically. This paper underscored that the process of vortex shedding is a very complicated one (see, for example, Smith 1985 or Tutty & Cowley 1986). Simple *ad hoc* modelling approaches are unlikely to be adequate.

The 'converse' problem of vortices impinging on solid bodies was studied by Yamada *et al.** who had produced flow visualizations and numerical simulations of a vortex pair colliding with a cylinder. Secondary vortices were spun-up in the boundary layer on the cylinder and with the assistance of their induced velocities the original pair 'rebounded'. Ishii, Lin & Kawahara* reported on studies of a vortex ring interacting with the boundary layer on a flat wall and in a pipe.

Van Atta, Gharib & Hammache* showed experimental results from wind-tunnel experiments on the shedding from a thin wire of circular cross-section. The shedding sets the wire vibrating and this leads to a rich variety of states where the vibration frequency and shedding frequency can be commensurate or incommensurate. Quasi-periodic and aperiodic (chaotic) time traces of velocity are observed, and using smoke visualization these are correlated with three-dimensional structures in the wake.

In many cases the control and reduction of boundary-layer separation and shedding is a prime objective. Modi *et al.** reported experiments on stalled airfoils with a leading edge consisting of a rotating cylinder on which separation could be virtually eliminated by rotating the cylinder at an appropriate speed. This type of control has been previously suggested by Viets (see, for example, Viets *et al.* 1981). Modi *et al.* had also considered multiple cylinder–airfoil combinations. The optimal positioning of a second cylinder is an intriguing problem. Applications to lorries could reduce drag by 17–20 % we were told.

Shed vortices are also responsible for the generation of *sound*. Since several papers were devoted to this topic, we shall consider it in a separate section.

5. Vortex sound

Küchemann (1965) is responsible for the oft-quoted characterization of vortices as the 'sinews and muscles of fluid motions'. Müller & Obermeier* in their survey suggested that vortices are also the 'voice' of the flow. Starting from Lighthill's (1952) theory they traced the evolution of the subject via the work of Ribner, Powell, Howe and Möhring to an explicit realization of the role of vorticity. They stressed that for compact sources all the different representations are equivalent and, thus, that it is attractive to work with a representation in which the vortex dynamics is immediately apparent. The formula for the pressure field in terms of an integral over the local time derivative of the vorticity field shows this connection explicitly: no vorticity, no sound. It then makes sense to consider the radiation from tractable models of vortex motion, and an example was shown of the radiation from an elliptical vortex ring calculated using a filament model. This calculation was prompted by experimental observations in an elliptic jet by Bridges & Hussain (1987), with which it qualitatively agrees, showing the applicability of this somewhat idealized model to 'real world' phenomena. The third-order time derivative of a quadrupole moment that appears in the expansion formula for the pressure in the far field exaggerates any intermittency of the vortex motion. Hence, the sound signal from the ellipse was much more jagged than the quasi-periodic nature of the motion might suggest. Müller & Obermeier* also examined the sound generated from vortex–airfoil interactions in transonic flow, and the scattering of sound by vortex flow (see figure 7). The former investigation was amplified in the companion paper by Meier, Lent & Löhr*. Zabusky commented that computational procedures now exist to perform simulation studies of this kind of problem that would compete favourably with the experimental resolution. Farge commented on the analogy between sound waves in the compressible-flow case and gravity waves in geophysical fluid dynamics.

Shariff *et al.** showed via numerical experiments that core deformations play an important role in explaining the time series of sound observed in vortex flows. A precipitous dip in the pressure seen when two coaxial rings run up against each other, which had been observed experimentally by Kambe & Minota (1983), could be explained by contour-dynamics simulations. Similarly, in computations of the passage of one ring through another oscillations in vortex shape were predicted to be responsible for additional fine-scale pressure fluctuations that could enhance the radiated power more than six times.

Minota, Kambe & Murakami* reported results and played tape recordings of the sound produced from vortex rings interacting with spheres, cylinders and an 'infinite' plane. The rings were 'shot' at the solid objects at different distances measured by an 'impact parameter'. As this impact parameter, and also the

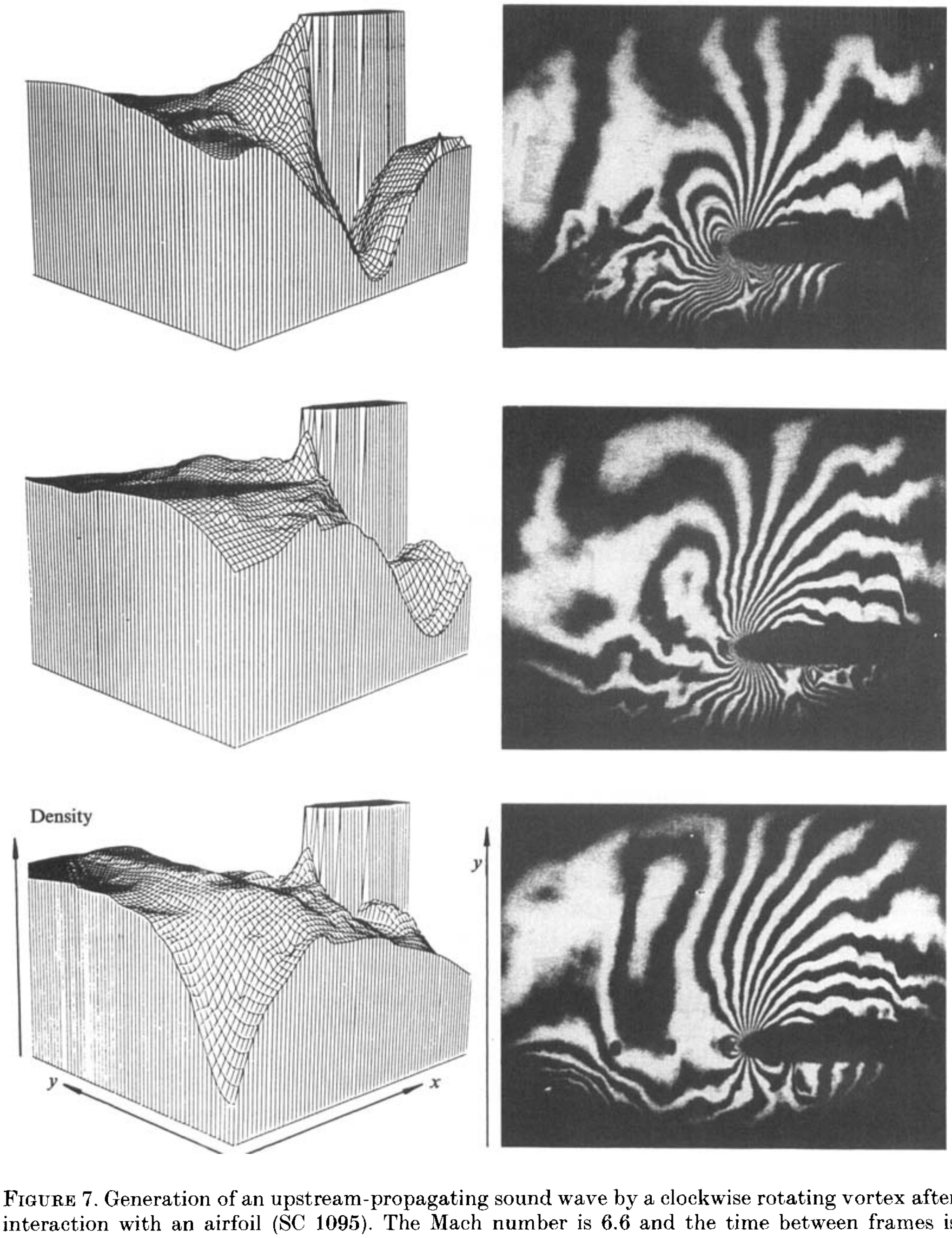

FIGURE 7. Generation of an upstream-propagating sound wave by a clockwise rotating vortex after interaction with an airfoil (SC 1095). The Mach number is 6.6 and the time between frames is 0.13 ms. In the right-hand column are interferometric observations, on the left, density calculations. From the paper by Meier, Lent & Löhr*.

direction of motion relative to the solid object, were varied the time trace of the sound would change, often quite substantially. A considerable amount of data now exists on these parametric dependencies.

Several papers dealt with the resonances that occur when shedding takes place in or close to a confining geometry. Mochizuki, Kiya & Tasumi* investigated the sound radiation from a jet impinging on a cylinder. Kawahashi, Brocher & Collini* considered the coupling between a small wedge shedding vortices upstream of a tube. Depending on the distance between the wedge and the mouth of the tube, the shedding pattern and period would be changed dramatically. The related papers by Mathias *et al.** and Thompson *et al.** dealt with the resonances observed in a wind-tunnel in which was placed a flat plate with semicircular leading edge. A loud resonant sound was produced by locking of the vortex shedding from the trailing edge of the plate to an acoustic mode of the tunnel. At lower flow velocities resonances were achieved at frequencies unrelated to the shedding frequency from the trailing edge. These were interpreted as sound radiated from vortices generated at the leading edge as they were swept past the trailing edge of the plate.

6. Flow topology

We have already alluded to the important role played in vortex dynamics by topological considerations and concepts, e.g. the helicity of the flow. Oil flow pictures of airfoils with separation clearly invite the systematic exploitation of topological concepts. Dallmann* traced how far one can go using essentially just the boundary conditions, ideas from topology and local bifurcation theory. Gibson* developed formulae for the motion of 'zero-gradient points' of a scalar such as temperature and of the enstrophy. He suggested scenarios for the evolution of curves and surfaces connecting these singularities with potential applications to turbulent mixing.

Topological considerations were a theme of many presentations. The vortex reconnection studies, for example, explicitly treated the 'unknotting' of a vortex. A scheduled contribution by Sym, that was unfortunately not presented, would undoubtedly have added more discussion on the topic of knotted vortices, and elucidated the N-soliton solution of LIA from a geometrical point of view (see the recent paper by Cieślinski, Gragert & Sym 1986). The paper by Meiburg, Lasheras & Ashurst*, a combined computational and experimental study of three-dimensional instabilities in wakes, traced the weaving of the two vortex arrays in a set of remarkably consistent plots from computer displays of the vorticity and flow visualization. The numerical experiments were done for inviscid flow, the laboratory experiments in the usual Kármán street regime. Depending on the initial perturbation, either in the plane of the mean flow or perpendicular to it, different patterns were produced. One was reminded of the two topologically distinct ways of knitting. A still tenuous relation to the instability patterns observed by Van Atta *et al.** was suggested.

Even in two dimensions there are important topological issues related to the degree of connectivity of certain regions (cf. figure 1). Dritschel* commented on an extension of contour dynamics that he and others have been pursuing in which the common interfaces of very close vortex patches are removed and thin 'necks' are cut (see Dritschel 1988*b*). This process, graphically referred to as *contour surgery*, amounts to a numerical truncation in the solution of the two-dimensional Euler equation (e.g. the circulation theorem is no longer valid for all material contours). It remains to be understood what the physical significance of contour dynamical solutions with surgery is. Are these solutions, for example, good models of the two-dimensional Navier–Stokes equation with small dissipation?

7. Concluding Remarks

There are invariably some papers at a meeting such as this one that do not fit conveniently into any of a few broad categories. A rigorous mathematical contribution by Giga & Kambe* established the asymptotic form of the vorticity field in two-dimensional flow decaying according to the Navier–Stokes equation and addressed vortex formation in three-dimensional flow. If a global Reynolds number, defined as the area integral of absolute vorticity divided by the kinematic viscosity, is small, these authors show that the asymptotic state is a single Oseen vortex regardless of the initial state. This was not a limit that had been much debated at the Symposium. No numerical estimate of how small the Reynolds number had to be for the result to hold was available.

Numerical results at the Symposium were many and varied, yet there was relatively little discussion of algorithms. The paper by van der Vegt* was one of the few departures from this rule. And the discussion that he gave was rather different from what is usually done in computational fluid dynamics in that it invoked a variational formulation, an action principle and other constructs from Lagrangian and Hamiltonian mechanics. The operator splitting into deterministic, inviscid dynamics and stochastic diffusion sounded familiar, but the formalism in which it was presented was very elegant and potentially powerful. Results were just beginning to flow from implementations of the procedures.

Pasmanter* pursued the correspondence between Lagrangian motion of a point in a flow and the 'phase space' motion of a dynamical system, a subject known as *chaotic advection* (Aref 1984). He was concerned primarily with particle dispersal by a time-dependent, two-dimensional model of tidal flow. The theme of chaotic advection was also briefly pursued by Shariff *et al.** who presented computed flow visualizations of two leap-frogging vortex pairs (see figure 8) that provide striking agreement with earlier flow visualization pictures by Yamada & Matsui (1978).

There were two presentations using entirely the framework of the statistical theory of turbulence. As we have already stated an underlying implicit assumption in much of vortex dynamics is that one can arrive at a mechanistic, deterministic understanding of turbulence phenomena, so the statistical aspects will arise as derivable consequences rather than necessary postulates. Nevertheless, these two contributions were strikingly relevant to the general discussion suggesting that one must worry about losing sight of the 'forest' by contemplating too many 'trees'. Nakano* worked within the framework of a 'cascade model' of turbulence, where wave-vector space is partitioned logarithmically into bands. Phenomenological equations are then written for the transport of energy from band to band. The main restriction on such model equations is that they capture an energy cascade in a reasonable way. Nakano* used this kind of model format to produce a scaling theory for the transient behaviour observed in turbulent decay. As is well known, vorticity dynamics is expected to play an important role here.

Tatsumi* started from the familiar equation for the evolution of the energy spectrum in homogeneous, isotropic turbulence (cf. Monin & Yaglom 1975). He then introduced as a working hypothesis the assumption of a wavenumber-independent dissipation rate, i.e. an equipartition principle. Dividing the wavenumber range into large and small scales he developed the effective equation for the large scales supposedly representing the coherent structures of the turbulence. The assumptions made allowed these equations to be cast in the form of inviscid hydrodynamics at the expense of a rescaling of the time variable. Hence, a closed form for the large scales

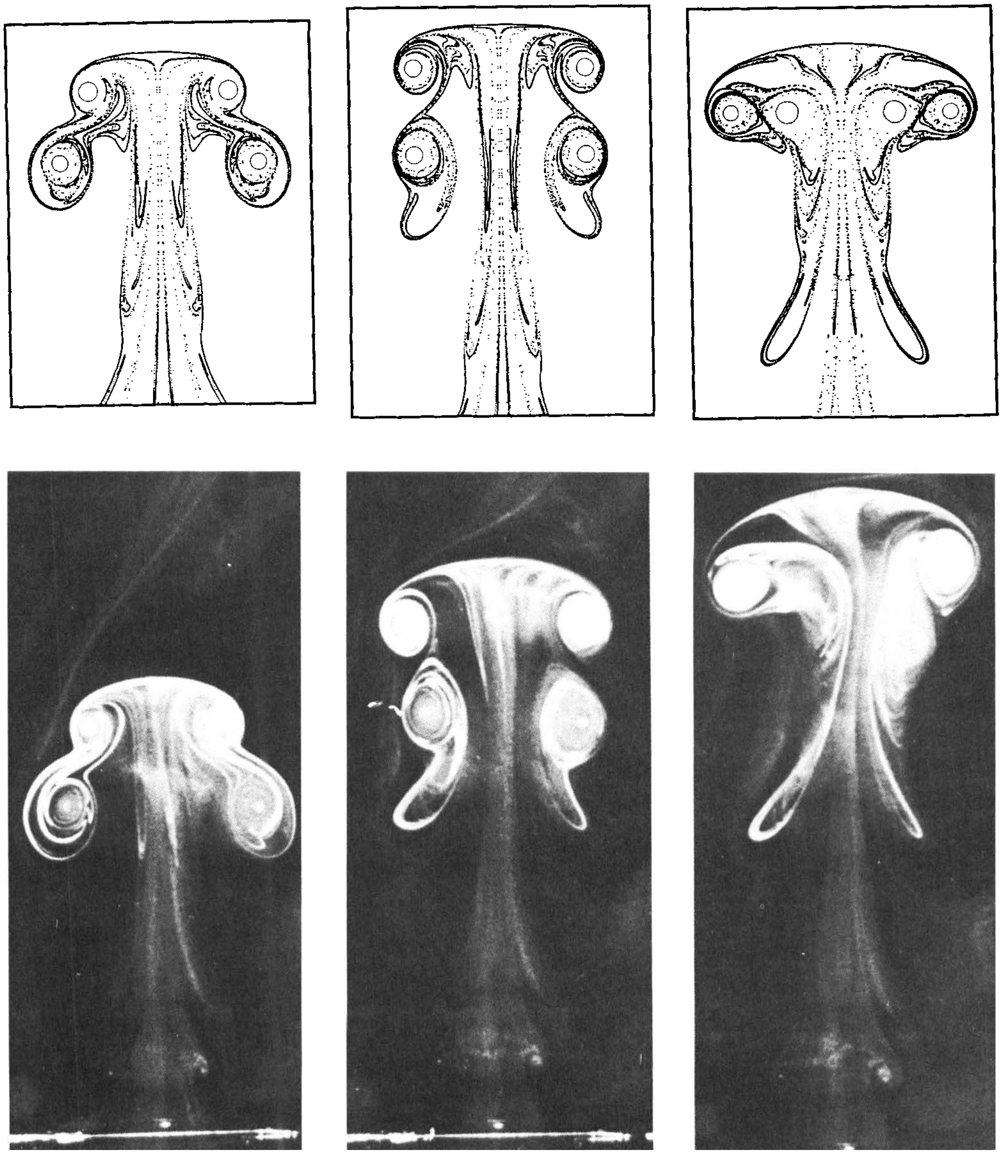

FIGURE 8. Chaotic advection of a marker in flow about two leap-frogging vortex pairs. The top row are from numerical simulations by Shariff et al.*, the bottom row are flow visualizations by Yamada & Matsui (1978). The vortices are shown by circles in the simulations, the marker particles are represented by dots. Note the elongated trailing lobes that contain no vorticity.

was obtained with the turbulent small scales acting principally to *slow down* the evolution. Applying this to a mixing layer, for example, it was possible to conclude that the exponential growth of instabilities in inviscid theory would correspond to a linear growth of eddies in the presence of small-scale turbulence! Simple statistical models of turbulent flow are usually controversial, but very often productive. One common criticism is that they implicitly assume a separation of scales that is not rigorously true in a turbulent flow. In contrasting such investigations to the detail provided by vortex dynamics one is reminded of the dichotomy between classical thermodynamics and kinetic theory.

What is new relative to the earlier symposium report by Küchemann (1965)? First, there are several new concepts: *chaos* and *solitons* were not widely used notions in fluid mechanics in 1965, and certainly not in the context of vortex motion. Second, a host of new phenomena such as merging, coupling, spiking, filamentation, tearing, bridging, etc. have been codified. Third, steady progress is being made on some thorny problems such as separation on delta wings and vortex breakdown. Fourth, the emphasis in turbulence research on *coherent structures* has in many ways put vortex dynamics at the heart of the 'turbulence problem'. Today we probably expect more insight into the physics of turbulence from deterministic models of vortex interactions than from purely statistical models. Finally, a surge of progress has taken place in computational studies of vortex motion (and most other aspects of physical science). Just to get a feeling of that progress the reader might compare the study by Abernathy & Kronauer (1962) to what is routinely done today 25 years later. Correspondingly, we were treated to several computer-generated films and videos both during the presentations and in separate movie sessions. In all more than a dozen different films were shown during the Symposium. One thing was unchanged: the number of presentations; there were just under 70 both at this Symposium and the one in 1965.

One gathers that the format of this Symposium was somewhat different than that of its precursor. Küchemann (1965) mentions a 15 minute time limit for the shortest presentations. At this Symposium there were six general lectures, each allotted an hour (including discussion), an opening lecture (45 minutes), and 30 oral presentations (25 minutes each). The remaining presentations were collected in so called 'visual sessions' in which an author was given 6 minutes to present a brief synopsis. There was then some additional time to view and discuss the poster presentation, video, and/or demonstration experiment. While Küchemann (1965) concludes that 'the salient points of most pieces of work can be communicated effectively in as short a time as 15 min., if the speaker is carefully prepared', this is not always true when the time interval is shortened to 6 minutes. Zabusky suggested that visual sessions in the future should be conducted in rooms full of computer graphics workstations, where participants could explore and discover the basic message of the presentation for themselves. However this prediction may turn out, there seem to be few substitutes for a coherent presentation of a body of research by an authority on that particular area, and there seems to be a lower bound on the amount of time necessary to set out a scientific explanation or argument so that it can be assimilated by an audience. With the increasing pace and volume of scientific publication, and the explosion in the number of meetings, it is difficult to make constructive recommendations. However, there is a danger that in the rush to be heard, the quality of the message is suffering to the detriment of the subject being so eagerly promoted.

The outlook of the Symposium participants may have been mostly academic but the topic of vortex motion has clear practical applications. This point was not lost on our Japanese hosts who had lined up an impressive list of private sector sponsors including the well-known major automobile manufacturers. The general nature of the topic of vortex motion made the Symposium an event of interest to mathematicians, physicists, meteorologists, oceanographers and engineers of all specialties.

We thank Drs Dritschel, Honji, Kida, Meier, Melander and Shariff for sending us figures and allowing us to reproduce them. We are indebted to Professors Maxworthy, Moffatt, Hussain, van Wijngaarden and Zabusky for comments on the manuscript. H.A. acknowledges support of NSF/PYI grant MSM84-51107 during the preparation of this report and for travel funds to attend the Symposium.

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